Mathematics for Computing - I

Sets (07 hrs.)

Instructional Objectives

- Illustrate properties of set algebra using Venn-diagrams.
- Prove various useful results of set algebra.

Main Reference

- Schaum’s Outline series: Discrete Mathematics, 2nd Edition by Seymour Lipshutz & Marc Lipson, Tata McGraw-Hill India, 2003, Chapter 01.

Introduction to Sets

- George Cantor (1845-1915), in 1895, was the first to define a set formally.
- Definition - Set

  A set is a unordered collection of zero of more distinct well defined objects.
- The objects that make up a set are called

  elements or **members** of the set.

Specifying Sets

- There are two ways to specify a set

1. If possible, list all the members of the set.

   E.g. A = {a, e, i, o, u}

2. State those properties which characterized the members in the set.

   E.g. B = {x : x is an even integer, x > 0}

   We read this as "B is the set of x such that x is an even integer and x is grater than zero". Note that we can't list all the members in the set B.

   C = {All the students who sat for BIT IT1101 paper in 2003}

   D = {Tall students who are doing BIT} – is not a set because "Tall" is not well defined. But…

   E = {Students who are taller than 6 Feet and who are doing BIT} – is a set.

Some Properties of Sets

- The order in which the elements are presented in a set is not important.
  - A = {a, e, i, o, u} and
  - B = {e, o, u, a, i} both define the same set.
- The members of a set can be anything.
- In a set the same member does not appear more than once.
  - F = {a, e, i, o, a, u} is incorrect since the element ‘a’ repeats.

Some Common Sets

- We denote following sets by the following symbols:
  – N = The set of positive integers = {1, 2, 3, …}
  – Z = The set of integers = {…,-2, -1, 0, 1, 2, …}
  – R = The set of real numbers
  – Q = The set of rational numbers
  – C = The set of complex numbers

Some Notation

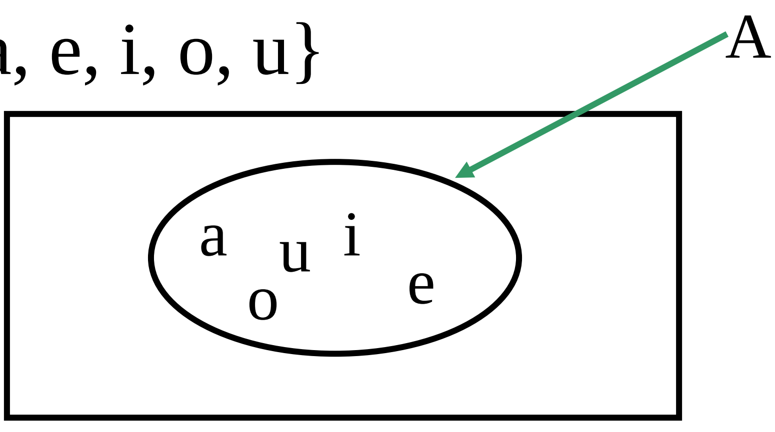
- Consider the set $A = \{a, e, i, o, u\}$ then
- We write "'a' is a member of 'A'" as:
  - $a \in A$
- We write "'b' is not a member of 'A'" as:
  - $b \notin A$
  - Note: $b \notin A \equiv \neg (b \in A)$

Universal Set and Empty Set

- The members of all the investigated sets in a particular problem usually belongs to some fixed large set. That set is called the universal set and is usually denoted by ‘U’.
- The set that has no elements is called the empty set and is denoted by Φ or {}.
  - E.g. $\{x \mid x^2 = 4 \text{ and } x \text{ is an odd integer}\} = \Phi$

Venn Diagrams

- A pictorial way of representing sets.
- The universal set is represented by the interior of a rectangle and the other sets are represented by disks lying within the rectangle.
  - E.g. A = {a, e, i, o, u}



Equality of two Sets

- A set 'A' is equal to a set 'B' if and only if both sets have the same elements. If sets 'A' and 'B' are equal we write: A = B. If sets 'A' and 'B' are not equal we write $A \neq B$.
- In other words we can say:

  $A = B \Leftrightarrow (\forall x, x \in A \Leftrightarrow x \in B)$

  – E.g.

    $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 1, 3, 5\}$, $C = \{1, 3, 5, 4\}$
    $D = \{x : x \in N \wedge 0 < x < 6\}$, $E = \{1, 10/5, \sqrt{9}, 2^2, 5\}$ then $A = B = D = E$ and $A \neq C$.

Cardinality of a Set

- The number of elements in a set is called the cardinality of a set. Let ‘A’ be any set then its cardinality is denoted by |A|
- E.g. A = {a, e, i, o, u} then |A| = 5.

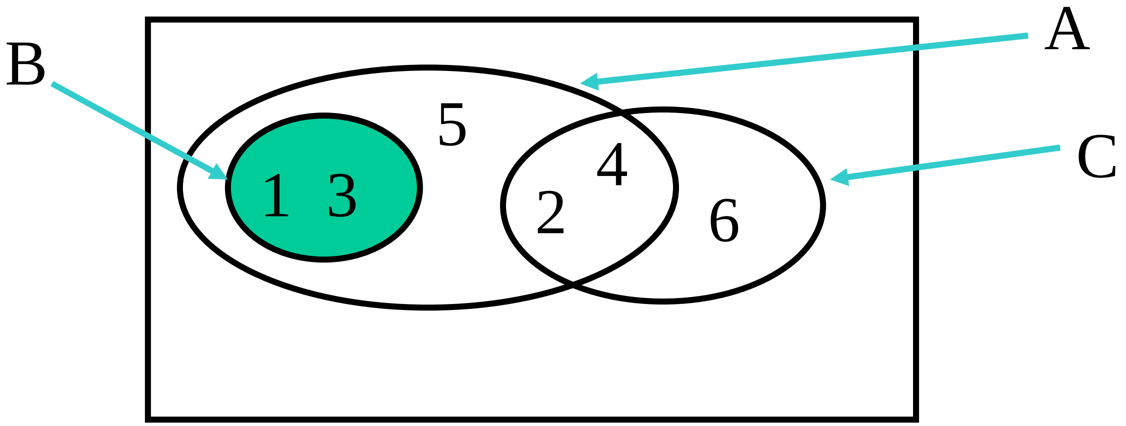
Subsets

- Set ‘A’ is called a subset of set ‘B’ if and only if every element of set ‘A’ is also an element of set ‘B’. We also say that ‘A’ *is contained in* ‘B’ or that ‘B’ *contains* ‘A’. It is denoted by $A \subseteq B$ or $B \supseteq A$.
- In other words we can say:

$$(A \subseteq B) \Leftrightarrow (\forall x, x \in A \Rightarrow x \in B)$$

Subset ctd…

- If ‘A’ is not a subset of ‘B’ then it is denoted by A $\nsubseteq$ B or B $\nsupseteq$ A
  - E.g. A = {1, 2, 3, 4, 5} and B = {1, 3} and C = {2, 4, 6} then B $\subseteq$ A and C $\nsubseteq$ A



Some Properties Regarding Subsets

- For any set 'A', $\Phi \subseteq A \subseteq U$
- For any set 'A', $A \subseteq A$
- $A \subseteq B \wedge B \subseteq C \Rightarrow A \subseteq C$
- $A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$

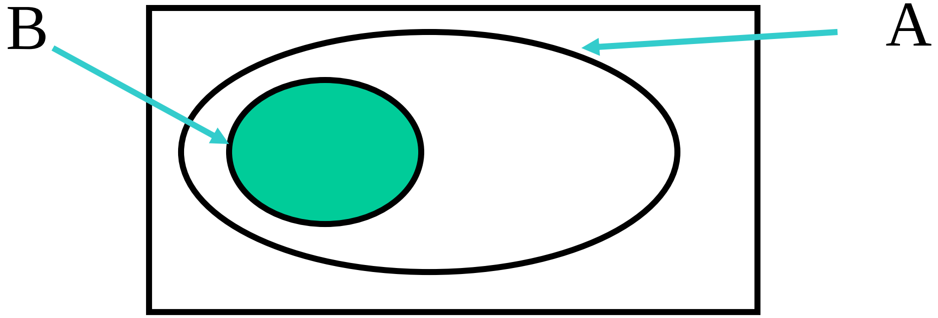
Proper Subsets

- Notice that when we say $A \subseteq B$ then it is even possible to be $A = B$.
- We say that set 'A' is a proper subset of set 'B' if and only if $A \subseteq B$ and $A \neq B$. We denote it by $A \subset B$ or $B \supset A$.
- In other words we can say:

$$(A \subset B) \Leftrightarrow (\forall x, x \in A \Rightarrow x \in B \wedge A \neq B)$$

Venn Diagram for a Proper Subset

- Note that if $A \subset B$ then the Venn diagram depicting those sets is as follows:



- If $A \subseteq B$ then the disc showing 'B' may overlap with the disc showing 'A' where in this case it is actually $A = B$

Power Set

- The set of all subsets of a set ‘S’ is called the power set of ‘S’. It is denoted by P(S) or 2^S.
- In other words we can say:

  $P(S) = \{x : x \subseteq S\}$

- E.g. $A = \{1, 2, 3\}$ then

  $P(A) = \{\Phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

- Note that $|P(S)| = 2^{|S|}$.
- E.g. $|P(A)| = 2^{|A|} = 2^3 = 8$.