IT2103

Mathematics for Computing 1

Logic

Algebra of Propositions

Propositions satisfy various laws. These laws are listed below.

1) Idempotent laws

   a) $p \vee p \equiv p$

   b) $p \wedge p \equiv p$

Algebra of Propositions ……

2) Associative laws

a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Algebra of Propositions ……

3) Commutative laws

a) $p \vee q \equiv q \vee p$

b) $p \wedge q \equiv q \wedge p$

Algebra of Propositions ……

3) Distributive laws

a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \vee r)$

Algebra of Propositions ……

4) Identity laws

a) $p \vee F \equiv p$

b) $p \wedge T \equiv p$

c) $p \vee T \equiv T$

d) $p \wedge F \equiv F$

Algebra of Propositions ……

5) Complement laws

a) $p \vee \sim p \equiv T$

b) $p \wedge \sim p \equiv F$

c) $\sim T \equiv F$

d) $\sim F \equiv T$

Algebra of Propositions ……

6) Involution law

a) $\sim\sim p \equiv p$

Algebra of Propositions ……

5) DeMorgan's laws

a) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

b) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Order of evaluation of logical expressions

Consider the following logical expression.

$$p \vee q \wedge r$$

This expression can be interpreted in two ways as below;

a) $(p \vee q) \wedge r$ b) $p \vee (q \wedge r)$

Question : Are these two expressions logically equivalent ?

Equality of $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$ can be determined by constructing truth tables for these propositions as below.

p	q	r	$p \vee q$	$q \wedge r$	$(p \vee q) \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	F	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

From the truth tables it is clear that $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$ are not logically equivalent. Therefore the meaning of the proposition $p \vee q \wedge r$ is ambiguous.

How can we disambiguate the meaning of such expressions ?

a) Defining precedence for logical connectives

~ has precedence over $\wedge$ which has precedence over $\vee$.

b) By using parenthesis.

Examples:

What is the evaluation of the expression $p \vee \sim q \wedge r$?

Solution :

The evaluation order of the expression is

$p \vee ((\sim q) \wedge r)$