# IT2103

# Mathematics for Computing 1

# Logic

# Types of Proof

1. Direct Proof

Let us prove that for any positive real number x, $x^3 + x > 0$.

Proof:

Suppose x is a positive real number; i.e. $x > 0$.

Then $x^3 + x = x(x^2 + 1) > 0$ (Since $x > 0$ and $x^2 + 1 > 0$).

Therefore for any positive real number x, $x^3 + x > 0$.

This is a direct proof.

Let us now look at an indirect proof.

# **Types of Proof continued…...**

(II)Indirect Proof.

Let us prove that for any real number x,
$x^3 + x > 0 \Rightarrow x > 0$.
We have to prove that when $x^3 + x > 0$ is true, then $x > 0$ is also true. We prove this indirectly. We assume that $x \in R$ and $x^3 + x > 0$ is true but $x > 0$ is false. We show that this cannot be so by obtaining a contradiction.From this we get, for any $x \in R$, if $x^3 + x > 0$ is true, then $x > 0$ is true.
Let us now give the proof.

# Types of Proof continued…...

Proof: Suppose $x \in R$ and $x^3 + x > 0$. Suppose $x > 0$ is false.
Then $x \leq 0$.
When $x = 0$, $x^3 + x = 0$. But $x^3 + x > 0$. Therefore $x \neq 0$.
Therefore $x < 0$ ( since $x \leq 0$ and $x \neq 0$).
Then, $x^3 + x = x(x^2 + 1) < 0$ ( Since $x < 0$ and $x^2 + 1 > 0$).
We get a contradiction ( Since $x^3 + x > 0$ and $x^3 + x < 0$).
Therefore, when $x \in R$ and $x^3 + x > 0$, $x > 0$.
Therefore, for any $x \in R$, $x^3 + x > 0 \Rightarrow x > 0$ //

A proof such as this is called a proof by contradiction.

# <u>**Types of Proof continued…...**</u>

We give another indirect proof of the above. It is called a <u>contrapositive</u> proof.

We have to prove that for any real number x, $x^3 + x > 0 \Rightarrow x > 0$.

Now we know that $p \Rightarrow q$ is equivalent to $(\sim q) \Rightarrow (\sim p)$. So we can prove the above by proving $x \leq 0 \Rightarrow x^3 + x \leq 0$.

This is called a <u>contrapositive</u> proof.

Let us now give the proof.

# Types of Proof continued…...

Proof: Suppose $x \in R$. Suppose $x \leq 0$. When $x = 0$, $x^3 + x = 0$.

Therefore when $x = 0$, $x^3 + x \leq 0$.

When $x < 0$ we have:

$x^3 + x = x(x^2 + 1) < 0$ ( Since $x < 0$ and $x^2 + 1 > 0$).

Therefore $x^3 + x < 0$.

Therefore $x^3 + x \leq 0$. /

Therefore, for any $x \in R$, $x \leq 0 \Rightarrow x^3 + x \leq 0$.

Therefore, for any $x \in R$, $x^3 + x > 0 \Rightarrow x > 0$.

# Counter Example

Consider the proposition, ‘for any positive integer n, 11n + 5 is not a prime number’. This proposition is false. To show this we give a counter example; i.e., we give an example of a positive integer n such that 11n + 5 is prime.

When n = 1, 11n + 5 = 16 is not prime. When n = 2, 11n + 5 = 27 is not prime. When n = 3, 11n + 5 = 38 is not prime. When n = 4, 11n + 5 = 49 is not prime. When n = 5, 11n + 5 = 60 is not prime. But when n = 6, 11n + 5 = 71 is prime.

So this is a counter example to

‘when n is a positive integer, 11n + 5 is not prime’.

# Counter Example continued…..

Let us consider again the above counter example. It is a counter example to the proposition '$\forall$n, 11n + 5 is not prime', where the universal set is N. By our counter example we show that, '$\forall$n, 11n + 5 is not prime' is false.

# Mathematical Induction

Consider a predicate P(n) where $n \in N$, for which $\forall n$ P(n) is true.
Then P(1) is true ------ (1).
Also, for any $n \in N$, $P(n) \Rightarrow P(n + 1)$ is true ------(2)
since for any $n \in N$, P(n), P(n + 1) are true.

Now, (1) and (2) are sufficient for $\forall n$, P(n) to be true.
Let us see why it is so.

# <u>Mathematical Induction continued….</u>

P(1) is true --------(1)

For any $n \in N$, $P(n) \Rightarrow P(n + 1)$ is true ------(2)

From (1) and (2), we get P(2) is true

(Since P(1) is true and $P(1) \Rightarrow P(2)$ is true)

Therefore, P(3) is true (Since P(2) is true and $P(2) \Rightarrow P(3)$ is true).

Therefore P(4) is true (Since P(3) is true and $P(3) \Rightarrow P(4)$ is true).

Proceeding this way we get for any $n \in N$, P(n) is true. This is

called the <u>principle of mathematical induction.</u>

# <u>Mathematical Induction continued…..</u>

We can apply the principle of mathematical induction to situations which are slightly different from the above situation. We give an example of this.

Consider the predicate $n^2 > 2n + 1$ where $n \in N$.
When $n = 1, 2$, $n^2 > 2n +1$ is false, but when $n \geq 3$, $n^2 > 2n + 1$ is true; i.e., $\forall n, n \geq 3 \Rightarrow n^2 > 2n + 1$ is true.
Let us prove this by the principle of mathematical induction.

# Mathematical Induction continued…..

When $n = 3$, $n^2 = 9$ and $2n + 1 = 7$.

Therefore when $n = 3$, $n^2 > 2n + 1$.

Suppose $n \in N$ and $n \geq 3$.

Suppose $n^2 > 2n + 1$.

Then, $(n + 1)^2 = n^2 + 2n + 1 > 2n + 1 + 2n + 1$ ( since $n^2 > 2n + 1$).

But $2n + 1 > 2$.

Therefore $(n + 1)^2 > 2n + 1 + 2$.

Therefore $(n + 1)^2 > 2(n + 1) + 1$.

# <u>**Mathematical Induction continued…..**</u>

For $n \in N$, let us denote $n^2 > 2n + 1$ by $P(n)$.

Then we have, $P(n) \Rightarrow P(n + 1)$ <u>whenever $n \in N$ and $n \geq 3$.</u>

So we have $P(3)$ is true ------(1)

and $P(n) \Rightarrow P(n + 1)$ whenever $n \in N$ and $n \geq 3$ ------(2).

From (1) and (2) we get $P(3)$, $P(4)$, $P(5)$…. are all true.

So, we have $\forall n, n \geq 3 \Rightarrow P(n)$ is true.