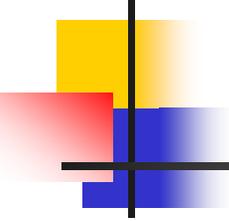


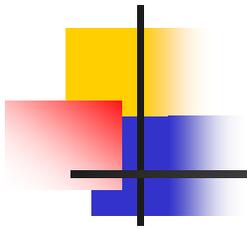
Techniques of Counting

Ms.M.Balamurali



Contents

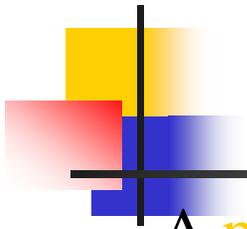
- Permutations
 - Introduction
 - Permutations with specific arrangements
 - Permutations with repetition
 - Practice with Permutation
- Combination
 - Introduction
 - Practice with combination
- Binomial theorem and Binomial Coefficients
- Pascal's triangle
- Pigeonhole Principle



Learning Outcomes

After completing this module students will be able to

- Calculate the number of elements in certain mathematically defined sets where ordinary methods of counting are tedious
- Calculate number of possible outcomes of elementary combinatorial processes such as permutations and combinations



Permutation

A **permutation** is an arrangement of objects in different orders.

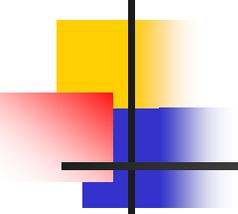
The order of the arrangement is important!! For example, the number of different ways 3 students can enter school can be shown as $3!$, or $3 \cdot 2 \cdot 1$, or 6 . There are six different arrangements, or permutations, of the three students in which all three of them enter school.

The notation for a permutation: ${}_n P_r$

n is the **total** number of objects

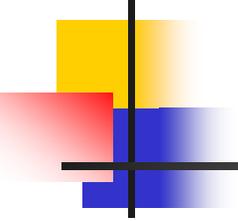
r is the number of objects chosen (want)

(Note if $n = r$ then ${}_n P_r = n!$)



Permutation

- To find the permutations of $\{A, B, C, D, E\}$, there are:
 - Five possible choices for the first item
 - Four possible choices for the second item
 - Three remaining possible choices for the third item
 - Two remaining possible choices for the fourth item, and
 - Only one possible “choice” for the final item
- For any positive integer N , we define $N!$ (“ N factorial”) as the product of all the positive integers up to and including N
 - Example: $5! = 1 * 2 * 3 * 4 * 5 = 120$
- Given any N *distinct* items, there are $N!$ possible permutations of those items



Permutation

By the rule of product,

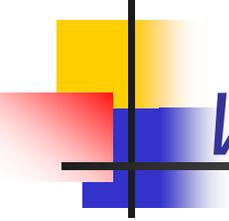
*The number of permutations of n things
taken r at a Time*

$$P(n, r) = n(n - 1)(n - 2) \dots (n - r + 1)$$

Note:

$$P(n, r) = \frac{n!}{(n - r)!}$$

Permutations



with Specific Arrangements

EXAMPLE:

Use the letters in the word " **square** " and tell how many 6-letter arrangements, with no repetitions, are possible if the :

- a) first letter is a vowel.
- b) vowels and consonants alternate, beginning with a consonant.

Solution:

Part a:

When working with arrangements, I put lines down to represent chairs.

Before starting a problem I decide how many chairs I need to fill and then work from there

I need six "chairs" (6-letter arrangements)

The first of the six chairs must be a vowel (u , a , e). There are three ways to fill the first chair.

3 · _____

After the vowel has been placed in the first chair, there are 5 letters left to be arranged in the remaining five chairs.

3 · 5 · 4 · 3 · 2 · 1 or

$$3 \cdot {}_5P_5 = 3 \cdot 120 = 360$$

Solution:

Part b:

I need six "chairs" (6-letter arrangements)

Beginning with a consonant, every other chair must be filled with a consonant. (s , q , r)

3 · _____ · 2 · _____ · 1 · _____

The remaining chairs have the three vowels to be arranged in them:

3 · 3 · 2 · 2 · 1 · 1

$$= 36$$

Permutations

with Repetition

In general, repetitions are taken care of by dividing the permutation by the number of objects that are identical!

Example 1:

1. How many different 5-letter words can be formed from the word **APPLE** ?

$$\frac{{}_5P_5}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{120}{2} = 60 \text{ words}$$

You divide by 2! Because the letter **P** repeats
twice

Permutations

with Repetition

Example:

2. How many different six-digit numerals can be written using all of the following six digits:

4,4,5,5,5,7

$$\frac{{}_6P_6}{2!3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = \frac{720}{12} = 60$$

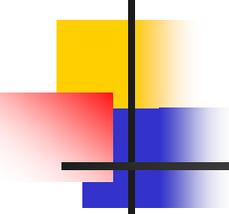
Two 4s repeat and three 5s repeat

Permutations

with Repetition

The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ..., and n_k indistinguishable objects of type k , is

$$\frac{n!}{n_1!n_2!\dots n_k!}$$



Combinations

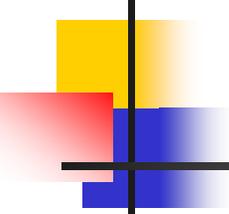
A **combination** is a set of objects in which order is *not* important.

*The number of combinations of n things
taken r at a time*

$$C(n, r) = \binom{n}{r} = \frac{P(n, r)}{r!} = \frac{n!}{(n-r)!r!}$$

or

$${}_n C_r = \frac{{}_n P_r}{r!}$$

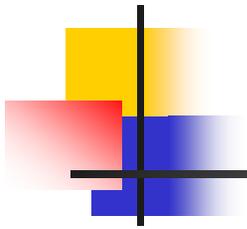


Combinations

Example1: Evaluate ${}^7C_2 = \frac{7 \cdot 6}{2 \cdot 1} = \frac{42}{2} = 21$

Example2: There are 12 boys and 14 girls in Mrs. Schultskie's math class. Find the number of ways Mrs. Schultskie can select a team of 3 students from the class to work on a group project. The team consists of 1 girl and 2 boys.

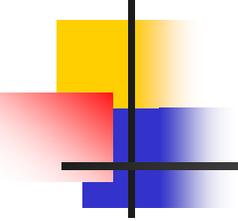
$$\begin{aligned} & \text{boy } {}_{12}C_2 \quad \text{girl } {}_{14}C_1 \\ & \frac{12 \cdot 11}{2 \cdot 1} \quad \frac{14}{1} \\ & 66 \cdot 14 \\ & = 924 \end{aligned}$$



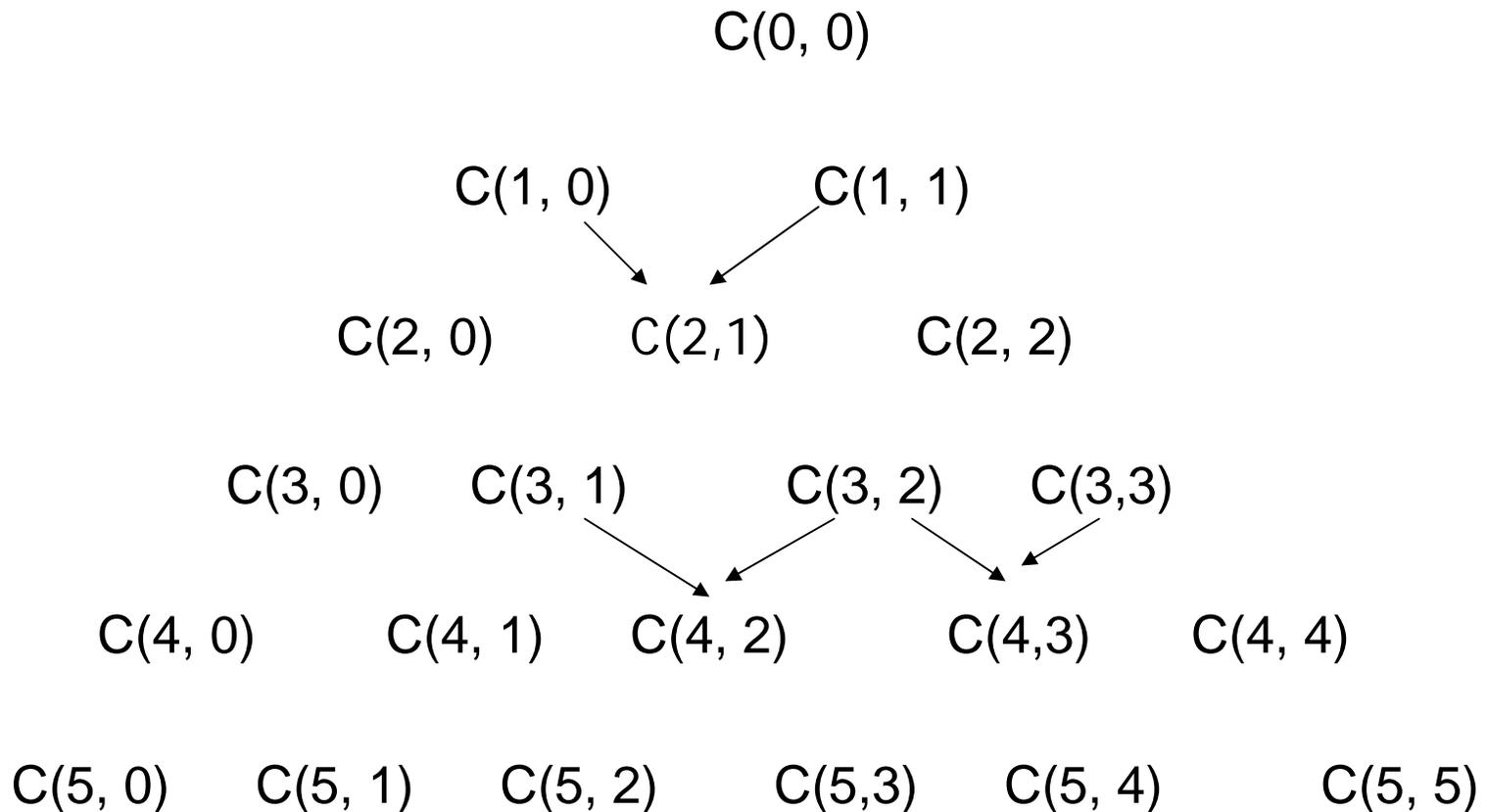
Binomial Theorem

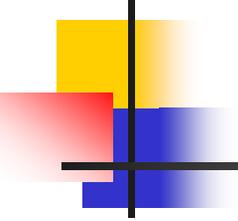
Let x and y be variables, and let n be a positive integer. Then

$$\begin{aligned}(x + y)^n &= \sum_{j=0}^n C(n, j)x^{n-j}y^j \\ &= \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n\end{aligned}$$

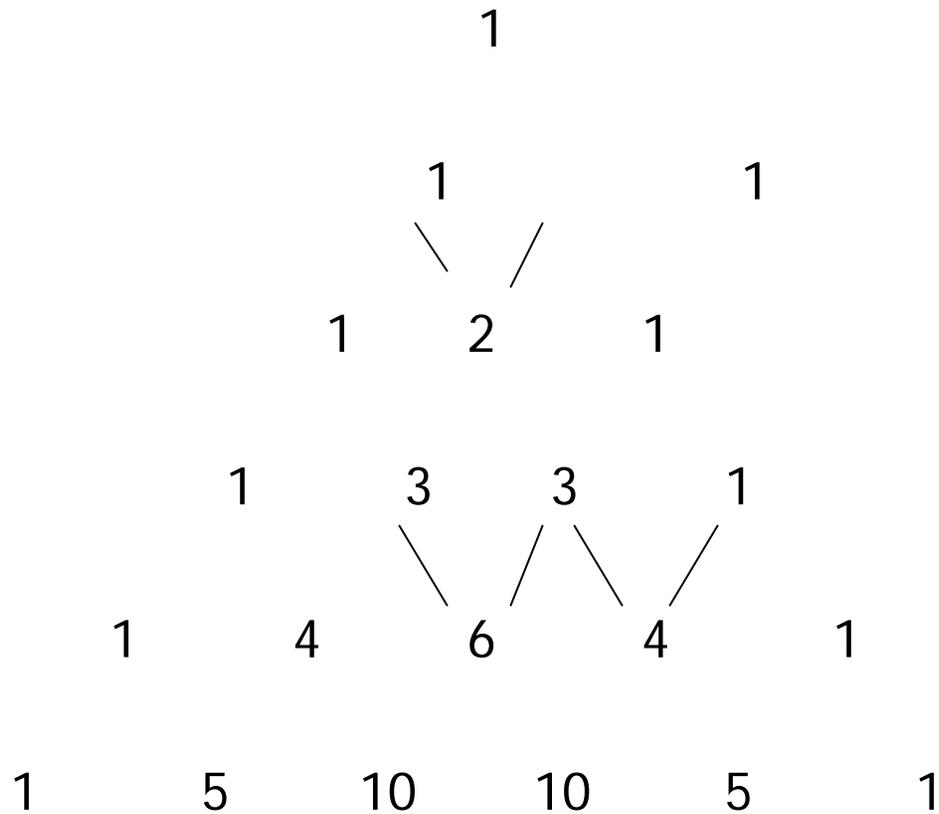


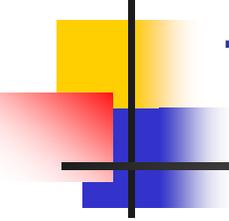
Pascal's Triangle





Pascal's Triangle





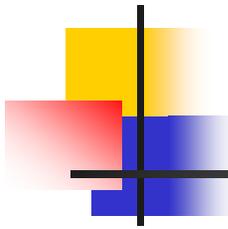
The Pigeonhole Principle

Pigeonhole principle

If n pigeon holes are occupied by $n+1$ or more pigeons, then at least one pigeon hole is occupied more than one pigeon.

Example:

Suppose a department contains 13 professors. Then two of the professors (pigeons) were born in the same month (pigeon holes).



The Pigeonhole Principle

Generalized pigeonhole principle

If n pigeonholes are occupied by $kn+1$ or more pigeons, where k is a positive integer, then at least one pigeon hole is occupied by $k+1$ or more pigeons.

Example:

Find the minimum number of students in a class to be sure that three of them are born in the same month.

Here the $n=12$ months are the pigeonholes and $k+1=3$, or $k=2$. Hence among any $kn+1=25$ students (pigeons), three of them are born in the same month.