IT2103

Mathematics for Computing 1

Logic

Arguments

Consider the following:

If prices are high, then wages are high. Prices are high or there are price controls. If there are price controls, then there is no inflation. There is inflation. Therefore, wages are high.

The above is an example of an argument.

Formally, an argument is a **sequence of propositions**. All propositions except the final one are called **premises** (or **assumptions** or **hypotheses**). The final proposition is called the **conclusion**. The symbol ∴ (or ⊢), read “therefore”, may be placed just before the conclusion.

Consider the following argument

If prices are high, then wages are high. Prices are high or there are price controls. If there are price controls, then there is no inflation. There is inflation. Therefore, wages are high.

In the above argument, we can refer to 'prices are high' by p, 'wages are high' by q, 'there are price controls' by r, and 'there is inflation' by s. Then the argument can be represented symbolically as below

$p \rightarrow q, p \vee r, r \rightarrow \sim s, s$

Therefore q.

An argument can be denoted in any of the following ways.

a) p_1, p_2, p_n
$\therefore$ q

b) $p_1, p_2, p_n \vdash q$

c) p_1, p_2, p_n
Therefore q

d) p_1, p_2, p_n
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q

In all the above representations $p_1, p_{2,\ldots}$ and p_n denote the premises of the arguments and q denotes the conclusion.

Validity of arguments

An argument is said to be **valid** if the conclusion of the argument is true whenever all premises are true. An argument which is not valid is called a **fallacy**.

How can the validity of an argument be determined ?

Example 1)

Consider the following argument

$p, p \rightarrow q$

Therefore q.

One of the ways to determine the validity of an argument is to construct its truth table and then determine the validity by consulting the truth table.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

From the above truth table it is clear that the conclusion (p) is true whenever all its premises (p, $p \rightarrow q$) are true. Therefore the argument is a valid argument.

Example 2)

Consider the following argument

$p \rightarrow q$, q

Therefore, p.

Is this argument valid?

Solution : To determine the validity of the argument construct the required truth table.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

From the above truth table it is clear that the both premises ($p \rightarrow q$ and q) are true only in two cases. For this argument to be valid, the conclusion (p) has to be true in both these cases. However the conclusion is not true in both cases. Therefore the argument is not valid. This means the argument is a **fallacy.**

However, when the number of propositions involved in an argument is large, it is not easy to construct the truth table for it. In such cases, the validity of the argument can be determined by analysing the argument.

Consider the following argument

$p \rightarrow q, p \vee r, r \rightarrow \sim s, s$

Therefore q.

In this argument 4 propositions are involved. If a truth table is to be built for the argument, 16 columns are needed to be constructed and this process takes substantial amount of time.

The validity of the argument can also be determined by analyzing it as below.

Suppose all the premises of the argument are correct.

- Then *s* is *T*.
- If *s* is *T*, ~*s* is *F*. Therefore $r \rightarrow \sim s$ to be true *r* has to be *F*.
- Since *r* is *F*, for $p \vee r$ to be true *p* has to be *T*.
- If p is true for $p \rightarrow q$ to be true **q has to be T**.

Thus it can be concluded that whenever all premises are T, the conclusion is also T. Therefore the argument is a valid argument.

Now consider the following argument:

If Anil's mark is more than 50, then his mark is more than 55. Anil's mark is more than 50 or his mark is less than 40. If Anil's mark is less than 40, then, Anil does not pass. Anil passes.

Therefore, Anil's mark is more than 55.

In the above argument refer to ‘Anil’s mark is more than 50’ by p, ‘Anil’s mark is more than 55’ by q, ‘Anil’s mark is less than 40’ by r and ‘Anil passes’ by s.

Then the argument is of the form:

$p \rightarrow q$, $p \vee r$, $r \rightarrow \sim s$, s

Therefore q.

We see that this is exactly the same as the earlier argument. Therefore we can conclude that this argument is also valid.